PRACE NAUKOWE Akademii im. Jana Długosza w Częstochowie
Pragmata tes Oikonomias 2013, z. VII

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Results of Monte Carlo Method Application to Performance Evaluation of Chosen Prioritization Procedures in the Analytic Hierarchy Process

Summary: The main goal of this research paper is to compare two procedures for preferences ranking operating within the Analytic Hierarchy Process, i.e. principal right eigenvector usually applied in this methodology and logarithmic utility approach, elaborated very recently as the alternative of the former one, with better proprieties. For examination purposes of this paper, Monte Carlo methodology was implemented with application of properly designed computer simulations. It can be noticed, that both procedures taken into consideration in this research are very competitive. Furthermore, the simulations results indicate, that the REV is not significantly better in comparison to the LUA in the process of deriving priority vectors from Pairwise Comparison Matrices. In fact, the REV and the LUA comparative evaluation leads to a conclusion that the REV, in some circumstances, may provide even worse than the LUA, estimation results of decision makers true priorities. Certainly, it is true for both types of PCMs, i.e. reciprocal and nonreciprocal. Moreover, the phenomenon observed in this research concerns the entire AHP model (not only single PCM). On the bases of this research, it seems reasonable to disagree with the concept widely distributed in the literature stating that the REV is the principal theoretical concept for deriving decision makers preferences and no other procedure qualifies. The main conclusion of the research paper is the fact, that in the Analytic Hierarchy Process, logarithmic utility approach is at least as good as principal right eigenvector procedure, and sometimes it prevails.
Keywords: AHP, preferences ranking, computer simulations, logarithmic utility approach, principal right eigenvector.

Introduction

Management is a process by which certain objectives are accomplished through the exploitation of different resources, e.g. energy, money, people, information, etc. These resources are considered to be inputs in this process, and the achievement of the goals is understood as the output of the process. The degree of success of a manager's performance is often measured by the ratio between outputs and inputs which defines the organization's productivity. As it determines the well-being of the organization and its members, it is considered as a major concern for any organization. The level of productivity, other words the success of management, depends and rely on the performance of certain managerial functions like planning, organizing, motivating and controlling. Certainly, to carry out these functions, managers are involved in a continuous process of making decisions.

Because all managerial activities revolve around this process we can agree that being a manager means firstly and foremost being a decision maker. For years, managers have considered decision making as a pure art, meant as talent acquired over a long period of time through experience, i.e. learnt by trial and error. It is known that a variety of individual styles can be used in approaching and successfully solving the same type of managerial problems within particular organization. These styles, as opposite to systematic quantitative methods based on a scientific approach, are often grounded on creativity, judgment, intuition, and experience. Maybe that is the reason why so important is to combine quantitative and qualitative decision making support systems while assisting and fostering managerial capabilities. Another, and maybe even more pertinent factor of this is growth of business complexity and its environment. This entails, it is more difficult to make decisions at the present times, and it is so for three reasons. Firstly, the number of alternative solutions for a problem is much larger today than ever before, mainly due to modern technology and communication systems. Secondly, future consequences of decisions are more difficult to foresee because of increased uncertainty. Finally, the cost of making errors may be very large due to the complexity and magnitude of operations, automation, and the chain reaction that potential error can cause in many parts of the organization. As a result of these trends and changes it would be very unreasonable to rely on a trial-and-error approach to management. Fortunately, we have some decision making support systems that are devised to facilitate us in making right choices. One of them is called the Analytic Hierarchy Process – AHP [8, 9].

1. Recognition of the problem

Generally, in order to make a decision, managers need various kinds of knowledge, information and technical data. These concern: details about the problem to be decided, the people or actors involved, their objectives and policies, the influences affecting the outcomes, the time horizons, scenarios and constraints [12]. As it occurs the AHP assists in systematization of these issues, although it is not just another analysis tool, and constitutes even more than just a methodology for choice (thousands of actual applications in which the AHP results were accepted and used by the competent decision makers), e.g. supplier selection [1, 10], energy selection [7], strategy selection [2], comparison of bridge designs [3], developing passenger train schedules [6], ranking university majors [11] or project complexity evaluation [15]). Thus, it can be considered to be both: a model of descriptive (relative measurement with distributive mode) and prescriptive (absolute measurement with ideal mode) decision making.

However, application of the AHP entails the concern about a consistency measurement of decision makers preferences expressed during implementation of this methodology. Certainly, there is a concept, developed together with the methodology itself, that enables to measure how inconsistent choices of decision makers are. Nevertheless, this concept works only with so called reciprocal Pairwise Comparison Matrices (PCMs), what means that it does not work with nonreciprocal PCMs, although their implementation unequivocally leads to better estimation results of decision makers preferences [4].

According to this concept the inconsistency of the data is measured as follows [14]. First a consistency index CI(n) is computed as an average of difference between $lambda_{max}$ and n for all eigenvalues of PCM except the principal one. Next, the value of the index is compared with an average random consistency index RI(n) obtained from a sample of 500 randomly generated reciprocal PCMs of order n. Finally, it is proposed to use so-called consistency ratio CR(n)=CI(n)/RI(n) for testing whether the information contained in the PCM is consistent enough to be acceptable. Unfortunately, besides this index is interpretable only for reciprocal PCMs it is also improperly constructed [4].

Nevertheless, there is recently devised procedure called Logarithmic Utility Approach to Eigenvector Method (LUA) that enables measurement of decision makers preferences consistency in both cases: reciprocal and nonreciprocal [9]. Then, the question appears if this procedure's performance in comparison to the Right Eigenvector Method (REV), the method applied by standards together with the AHP, is credible enough it could be used instead. In order to analyze this problem in more details we will proceed with properly designed simulations with the application of original Saaty's scale.

2. Scenario description with results of Monte Carlo simulation

The simulation scenario we proceed involves the entire AHP model that consists of three levels: goal, criteria and alternatives. The assumption is the scenario should reflect the hypothetic case of real decision problem being considered under the auspices of AHP. Thus, in order to compare the performance accuracy of the LUA and the REV we simulate different situations related to various sources of the PCMs inconsistency.

The scenario can be worded as follows. We generate uniformly random and normalized 'true' priority vector (TPV) for uniformly randomly chosen number of criteria (n_c). Next, we generate uniformly random and normalized 'true' priority vectors for the given set of criteria with uniformly randomly chosen number of alternatives (n_a), e.g. for one Priority Vector (PV) with three criteria we generate three PVs with chosen number of alternatives. Then, we calculate the 'true' total priority vector of weights (TTPV), according to a well-prescribed procedure (standard AHP aggregation based on weighting and adding). Next, on the bases of 'true' priority vectors (TPVs) generated for the given set of criteria and the given sets of alternatives, we create correspondent pairwise comparison matrices (PCMs). Then, we make them inconsistent through perturbation of their elements in accordance with the relation (1):

$$a_{ij} = e_{ij} \cdot w_{ij} \tag{1}$$

where e_{ij} is a perturbation factor near one with a given probability distribution.

Next, on the bases of such created inconsistent PCMs we compute their respective priority vectors with the application of the LUA and the REV. Then, for each prioritization procedure (LUA & REV) we calculate its respective total priority vector (TPV_{LUA} and TPV_{REV}) applying standard AHP aggregation procedure. Finally, we compare such obtained results with the values of original TTPVs.

In order to evaluate the performance of chosen methods, we compute known from literature [5] the Pearson Correlation Coefficients r between the 'true' total priority vectors (TTPVs) and their estimates due to the application of LUA and REV procedure, i.e. TPV_{LUA} and TPV_{REV}, their Spearman Rank Correlation Coefficients ρ, and Mean Absolute Deviations (MAD). In our research, we always consider two approximation options: with and without forced reciprocity. When forced reciprocity condition is applied to PCM, the perturbed PCM inputs are taken only from above its diagonal elements, and the remaining ones are entered as the inverses of the corresponding symmetric units in relation to its diagonal elements. The PCMs with forced reciprocity condition applied are denoted as FRPCMs. Nonreciprocal PCMs in this article are denoted as APCMs.

The number of criteria and alternatives in the particular AHP model is uniformly drawn from the set N = {4, 5, 6, …, 12} separately for criteria and alternatives. We examine one thousand such AHP models for each perturbation factor probability distribution scenario (uniform, log-normal, truncated normal or gamma), and perturb every different AHP model with perturbation factor only once. In conclusion this scenario gives us the performance measures for one thousand cases. Every time the perturbation factor e_{ij} is uniformly, log-normally, truncated normally or gamma drawn, and it is drawn from the following assigned interval: $e_{ij} \in [0,5;\ 1,5]$. The performance results of LUA and REV under just described simulation framework are going to be presented now (tables 1, 2, 3, 4).

Table 1. Performance evaluations of the LUA and the REV for 1000 cases within uniformly drawn AHP framework: n_c, $n_a \in \{4,\dots, 12\}$ and $e_{ij} \in [0,5; 1,5]$ with gamma probability distribution

Performance measures		ρ	r	MAD
APCM	REV	0,924520	0,991800	0,00957736
	LUA	0.924351	0.991678	0,00964557
FRPCM	REV	0,891378	0,980005	0,01485830
	LUA	0,893134	0,980570	0,01472490

Source: own elaboration.

Table 2. Performance evaluations of the LUA and the REV for 1000 cases within uniformly drawn AHP framework: n_c, $n_a \in \{4,\dots, 12\}$ and $e_{ij} \in [0,5; 1,5]$ with log-normal probability distribution

Performance measures		ρ	r	MAD
APCM	REV	0,934059	0,992418	0,00921376
	LUA	0,932632	0,992303	0,00930053
FRPCM	REV	0,917239	0,985688	0,01280100
	LUA	0,917174	0,985835	0,01275180

Source: own elaboration.

Table 3. Performance evaluations of the LUA and the REV for 1000 cases within uniformly drawn AHP framework: n_c, $n_a \in \{4,\dots, 12\}$ and $e_{ij} \in [0,5; 1,5]$ with truncated normal probability distribution

Performance measures		ρ	r	MAD
APCM	REV	0,979448	0,999174	0,00309905
	LUA	0,979517	0,999166	0,00310904
FRPCM	REV	0,977246	0,998918	0,00359482
	LUA	0,977246	0,998919	0,00359457

Source: own elaboration.

Table 4. Performance evaluations of the LUA and the REV for 1000 cases within uniformly drawn AHP framework: n_c, $n_a \in \{4,\dots, 12\}$ and $e_{ij} \in [0,5; 1,5]$ with uniform probability distribution

Performance measures		ρ	r	MAD
APCM	REV	0,963286	0,997053	0,00559876
	LUA	0,962726	0,997022	0,00563455
FRPCM	REV	0,955477	0,995495	0,00691381
	LUA	0,955481	0,995503	0,00691094

Source: own elaboration.

Conclusion and final remarks

It can be noticed, that both procedures taken into consideration in this research are very competitive. Furthermore, the simulations results indicate, that the REV is not significantly better in comparison to the LUA in the process of deriving priority vectors from Pairwise Comparison Matrices. In fact, the REV and the LUA comparative evaluation leads to a conclusion that the REV, in some circumstances, may provide even worse than the LUA, estimation results of decision makers true priorities. Certainly, it is true for both types of PCMs, i.e. reciprocal and nonreciprocal. Moreover, the phenomena observed in this research concerns the entire AHP model (not only single PCM). On the bases of this research, it seems reasonable to disagree with the concept widely distributed in the literature [13] stating that the REV is the principal theoretical concept for deriving decision makers preferences and no other procedure qualifies.

To recapitulate, it is justifiable on the grounds of this research to state that the LUA in the AHP is at least as effective as the REV and sometimes it even prevails. It is so because the Logarithmic Utility Approach to the Eigenvector Method can be applied to both reciprocal and nonreciprocal PCMs, it is computationally simpler, it allows decision makers for introduction of additional constraints which cannot be applied to the REV, it does not violate the condition of order preservation [9] and finally it provides intuitive and uncomplicated measure of consistency then, when the REV fails (nonreciprocal PCMs). This all together provides a relatively simple yet still very powerful prioritization technique for very unsophisticated multicriteria decision making support methodology considered as most widely used in the world today.

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Wyniki zastosowania metody Monte Carlo do oceny funkcjonowania wybranych procedur rangowych w Procesie Analitycznej Hierarchii

Synopsis: Niniejsza praca badawcza ma na celu porównanie funkcjonowania dwóch procedur szacowania rang w Procesie Analitycznej Hierarchii, tj. metody wektora własnego, stosowanej standardowo w tym procesie, i podejścia użyteczności logarytmicznej, opracowanego stosunkowo niedawno jako procedury alternatywnej o lepszych własnościach. Do celów badawczych zastosowano metodę Monte Carlo, przy wykorzystaniu jako narzędzia badawczego stosownie zaprojektowa-

nych symulacji komputerowych. Można zauważyć, że obydwie procedury rozpatrywane w pracy badawczej są bardzo konkurencyjne. Ponadto, jak wykazały wyniki symulacji komputerowych, z punktu widzenia procesu wywodzenia wektora rang z macierzy elementów porównywanych parami, metoda wektora własnego nie jest znacząco lepsza w porównaniu do podejścia użyteczności logarytmicznej. Można też zauważyć, że w pewnych okolicznościach używanie metody wektora własnego do procesu estymacji prawdziwych preferencji decydentów prowadzi nawet do gorszych efektów. Podkreślić wypada także, że stwierdzenie to jest prawdziwe zarówno dla macierzy konstruowanych z elementów symetrycznych względem jej głównej przekątnej, jak również tych niesymetrycznych. Co więcej, jest prawdziwe również dla całego modelu AHP, a nie tylko pojedynczej macierzy. Na podstawie niniejszej pracy badawczej można więc nie zgodzić się ze stwierdzeniem opublikowanym w literaturze, głoszącym, że metoda wektora własnego jest główną procedurą wywodzenia preferencji decydentów i żadna inna się nie kwalifikuje. Głównym zatem wnioskiem tego artykułu jest stwierdzenie, że podejście użyteczności logarytmicznej jest przynajmniej tak samo efektywne w Procesie Analitycznej Hierarchii, jak metoda wektora własnego, a czasami efektywniejsze.
Słowa kluczowe: Proces Analitycznej Hierarchii, szeregowanie preferencji, symulacje komputerowe, podejście użyteczności logarytmicznej, główny prawy wektor własny.