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The concept of resilience and its application to a certain socio-economical model

1. Introduction

In recent decades, there is a growing interest in ecological and social systems that exhibit multistability, i.e., having alternative stable states. The early hints, that real ecosystems and socioeconomical systems can exhibit such properties, originated from theoretical models [8, 13]. Although regime shifts are very easy to show and explore in models, it took time and efforts to prove experimentally that such phenomena occur in the real world [15, 20, 22]. Moreover, manipulation experiments have also provided direct evidence for alternative stable states [21]. Probably the most famous case of a bistable system is the case of alternative equilibria in shallow lakes [20]. Over certain ranges of nutrient concentration shallow lakes have two alternative equilibria: a clear state dominated by macrophytic aquatic vegetation, and a turbid state with high algal biomass.

A dangerous property of bistable systems – dangerous at least from the point of view of its human participant – may be so-called hysteresis effect. The state of the system depends not only on the current circumstances but also on the history of the system itself. As for the shallow lakes that means, that if the nutrient concentration in the lake is growing it may in some point, call it B, lead to a rapid regime shift – from clear state to turbid state. What worse, if we now want to return to the clear state it is not sufficient to lower nutrient concentration to the level B, but to even lower level, A: $A < B$ and in general is not know in advance.

The great importance is then in recognizing the real state of the system, that is, “how far” from the regime shift it is. Holling [8] has introduced a definition of *resilience*, as ”a measure of the ability of systems to absorb changes of state variables, driving variables, and parameters, and still persist”

The above definition of resilience does not determine how to measure this quantity in a mathematical manner. This is a question of great practical importance: how can one examine the resilience properties of a real system, and how can one predict whether the system will persist in the face of external stresses or shocks or survive natural fluctuations? How can we measure resilience in order to forecast system's dynamics and manage or prevent the consequences from its breakdown, particularly when it declines to levels where state shift becomes inevitable?

In this paper we will analyze some of the most commonly used measures of resilience. They will be applied to the simple socio-economical model – binary choice model of Brock and Durlauf [3,5], and compared.

2. Brock-Durlauf model.

In the general framework of binary-choice models there exists a widely discussed Brock-Durlauf model [3, 5, 17]. Here we present a dynamical version of it. The model describes a set of N individuals, each of them faced to a repeating choice. At each time step an individual has to choose (+1) or (−1), depending on relative gains and losses upon each possible choice. The choice of an individual i in time step t will be denoted by s_i^t: $s_i^t \in \{-1,1\}$ Gains and losses are evaluated using a function containing external influences, mutual interactions among individuals, and a random term:

$$s_i^t = \arg\max_{s_i^t \in \{-1,1\}} U_i\left(s_i^t\right) \tag{1}$$

where:

$$U_i\left(s_i^t\right) = h_i s_i^t + s_i^t \sum_{j \neq i} \frac{J_{ij}}{2} s_j^{t-1} + \epsilon\left(s_i^t\right). \tag{2}$$

Here h_i denotes external influences (e.g. external profit), J_{ij} – strength of interaction between individuals i and j and $\epsilon\left(s_i^t\right)$ is a random term (realizations of random variables $\mathcal{E}(-1)$ and $\mathcal{E}(+1)$ independent for each time step). Note that, as the choice is undertaken upon the difference, $U_i(-1) - U_i(+1)$, only the difference $\epsilon(-1) - \epsilon(+1)$ plays the role in decision making. The random variable ξ, being the difference $\mathcal{E}(-1) - \mathcal{E}(+1)$, will be taken here to be logistic one:

$$\xi \equiv \mathcal{E}(-1) - \mathcal{E}(+1)$$

$$P(\xi < z) = \frac{1}{1+\exp[-\beta z]}, \tag{3}$$

where β is a parameter that determines the width of probability density distribution. The bigger value of β the more deterministic the system is (as probability of $\xi \neq 0$ tends to 0 with $\beta \to \infty$).

Let us assume, that each individual interacts with all the others with the same strength (that is, $J_{ij} \equiv J$), and that the values of parameters are uniform across the system ($h_i \equiv h$). Let us also define a mean choice, m, as:

$$m^t = \frac{1}{N}\sum_i s_i^t.$$

We are interested here in stationary states of the model, that is, such states within which mean choice does not change in time,

$$m^t = m^{t-1}, \tag{4}$$

as only such states can persist for time long enough to be observed. Any other state will immediately evolve toward one satisfying (4). In what follows we will discuss continuous-time approximation.

The continuous-time version of the model is described by a differential equation:

$$\frac{dm}{dt} = \tanh[\beta(h + Jm)] - m, \tag{5}$$

and stationary state condition becomes:

$$\frac{dm}{dt} = 0. \tag{6}$$

Mean choice within stationary state, that is, fulfilling (6), will be denoted by m^*. From (5) and (6) it follows:

$$\tanh[\beta(h + Jm^*)] = m^*. \tag{7}$$

In the case of $\beta J < 1$ there always exists only one solution, see Fig.1a. However, under the condition $\beta J > 1$ there exists some range of h values within which the condition (7) has three solutions, as shown in Fig.2a. The solutions are pictured on the left parts of Figs.1,2 as intersections of the curve $F(m) \equiv \tanh[\beta(h + Jm)]$ and the straight line m. In Fig. 1a $F(m)$ is plotted for $h = 0$ (solid line), $h = +0.2$ (dotted line) and $h = -0.2$ (dashed line). In Fig.2a three solutions exist for $|h|$ sufficiently small. $F(m)$ is plotted there for $h = 0$ (solid line), $h = +0.5$ (dotted line) and $h = -0.5$ (dashed line). Then, Let us fix the values of β and J: $\beta = 2$, $J = 1$. A stationary state with $m < 0$ exists for $h \in (-\infty, +0.2664)$, while two stable stationary states exist for $h \in (-0.2664, +0.2664)$, and a stationary state with $m > 0$ exists for $h \in (-0.2664, +\infty)$. This produces a hysteresis of width $2 \cdot 0.2664 = 0.5328$, cf. dotted line on Fig.3. Generally, the bigger value of β the wider hysteresis, cf. Fig.3, where three hysteresis for $J = 1$ and $\beta = 2$ (dotted line), $\beta = 5$ (dashed line) and $\beta = 50$ (solid line) are shown.

For illustrativeness, let us introduce a concept of *potential*. For continuous systems, described by a differential equations in the form: $\frac{d\bar{X}}{dt} = \bar{F}(\bar{X})$, the potential is defined as:

$$V(\bar{X}) = \int \bar{F}(\bar{X}) d\bar{X}. \tag{8}$$

The conditions for a stationary stable point, denoted by $\bar{X}^*$ are:

$$\bar{F}(\bar{X}^*) = 0, \quad \left.\frac{d\bar{F}(\bar{X})}{d\bar{X}}\right|_{\bar{X}=\bar{X}^*} < 0,$$

what implies, that for stable stationary point such defined potential has minimum. Thus, the system can be considered as a ball rolling in the cup whose walls take a shape defined by that potential, always tending towards the bottom.

According to definition (8) the potential for Brock-Durlauf model described by (5) reads:

$$V_{BD}(m) = \frac{m^2}{2} - \frac{1}{\beta J} \ln \cosh[\beta(h + Jm)]. \tag{8}$$

The analysis of existence of stationary states performed two paragraphs above may be here understood in terms of the shape of potential. For $\beta J < 1$ the potential has always only one minimum, regardless the value of h. Right hand side of Fig.1 shows the shape of potential for these same values of parameters as on the left hand side, and right hand side of Fig.2 shows the shape of potential for these same values of parameters as on the left hand side of this figure. It may be seen, that existence of three solutions corresponds to double-well potential (two solutions corresponding to two minima, while the third solution corresponds to the unstable stationary state, i.e. maximum of potential). As for hysteresis, in the range of hysteresis loop the potential has two minima, while out of that range one of the minima vanishes. Figure 4 shows changes of shape of the potential and the state of the system while changing value of h (for $J = 1$ and $\beta = 50$). It can be seen that, approaching a certain value of h one of the wells becomes more and more shallow and finally vanishes.

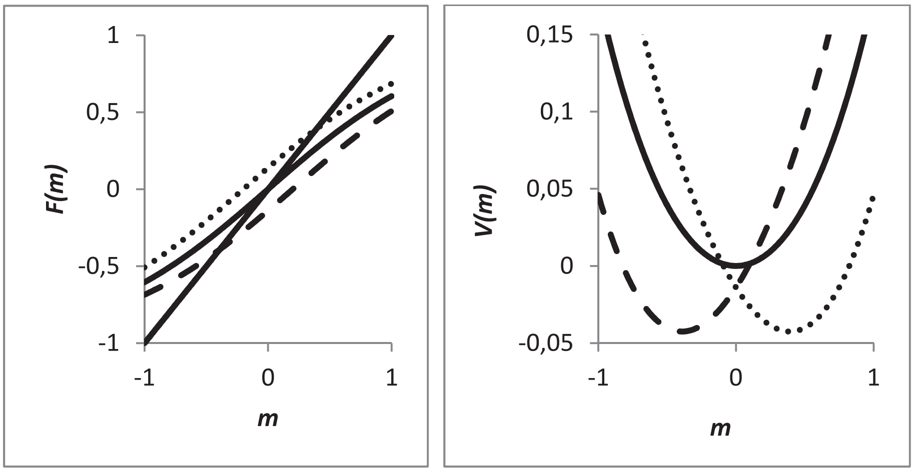

Fig.1. Solutions of stationary state condition (a) and shape of the potential (b) for $\beta J < 1$.

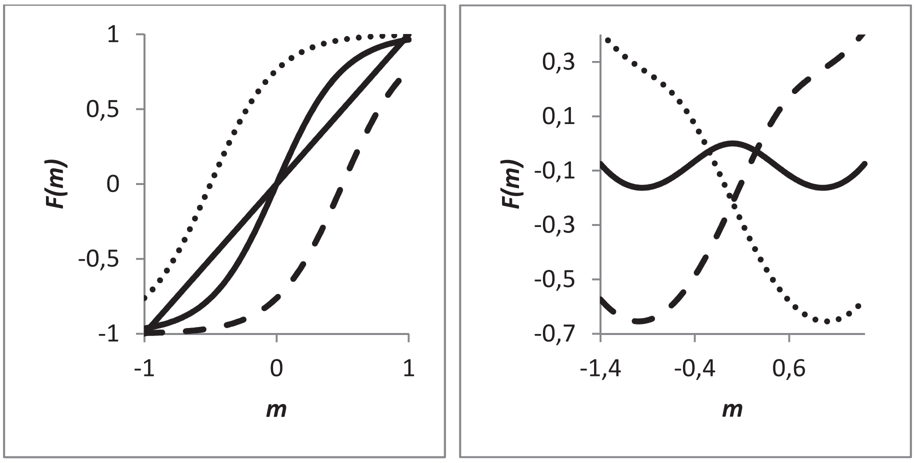

Fig. 2. Solutions of stationary state condition (a) and shape of the potential (b) for $\beta J > 1$.

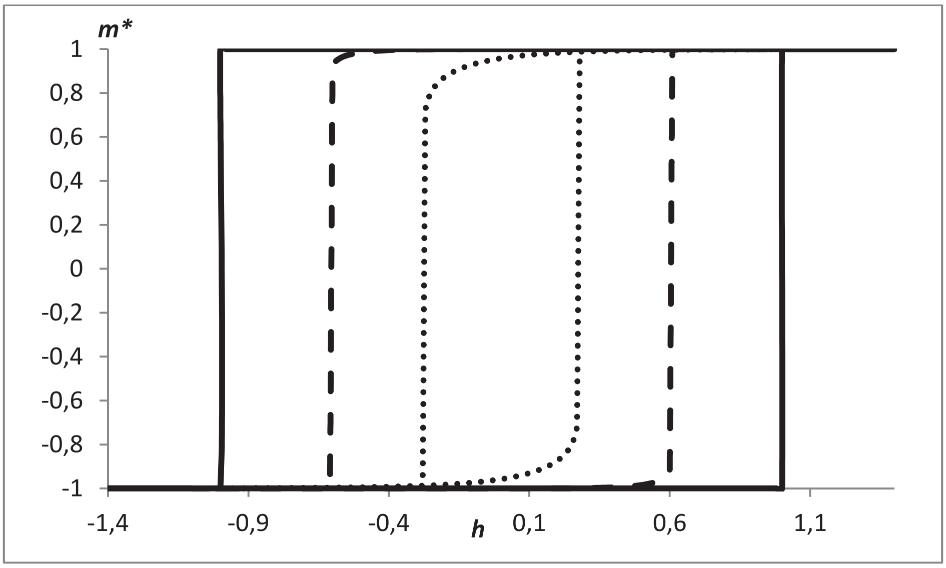

Fig. 3. Hysteresis in Brock-Durlauf model.

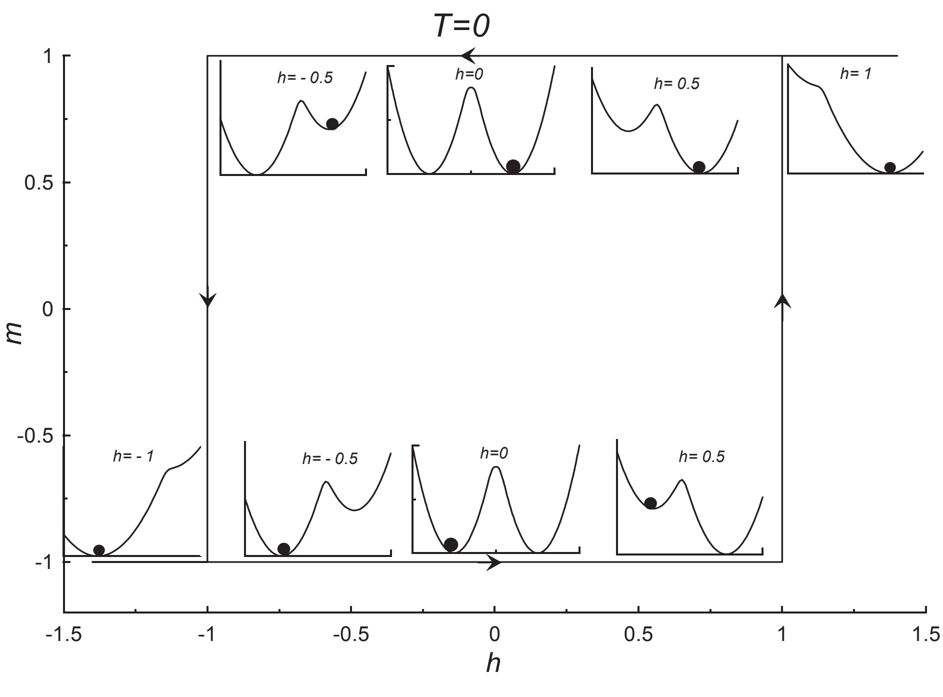

Fig. 4. Shape of potential and state of the system in various points of hysteresis.

In next section we will proceed to introduce some best known measures of resilience which be than applied to the Brock-Durlauf model.

3. Some measures of resilience applied to Brock-Durlauf model

According to Holling's definition there are two measures of resilience of a given state corresponding to the minimum of potential and based on the concept of it [7, 8]. Firstly, the overall area of the domain of attraction of this state, which in one dimension reduces simply to the width of well. Secondly, the height of the "potential barrier" that separates the basins of attraction of different regimes (different wells), see Fig. 5. The former corresponds to the maximum perturbation of a state parameter (e.g., an instant mortality event) and the latter to the maximum perturbation of a driving force (e.g., caused by a temperature peak). Both of these quantities have to be considered jointly to establish a proper value of resilience of a given system.

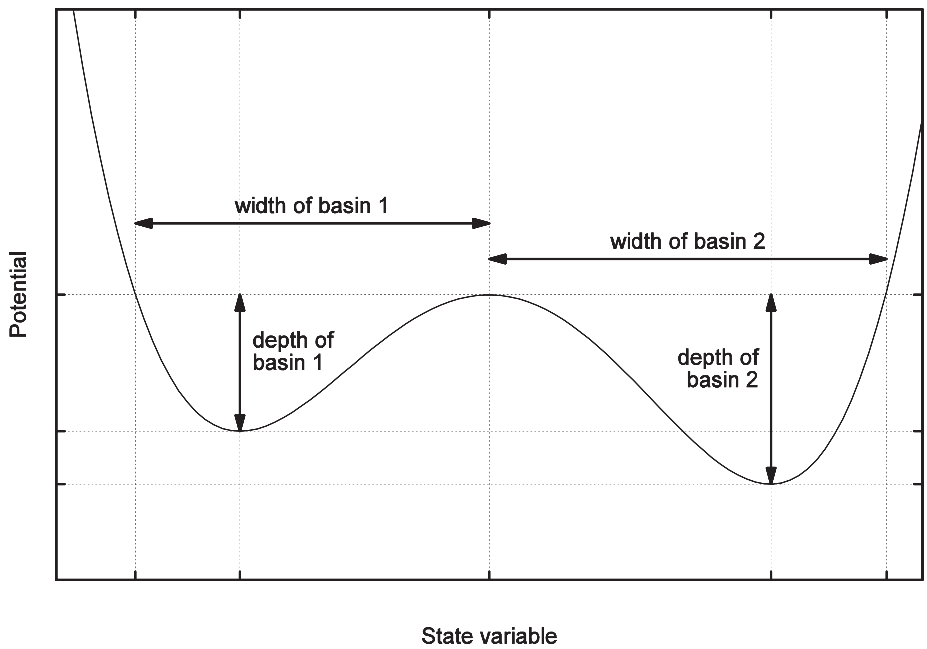

Fig. 5. Measures of resilience for one dimensional potential.

The shape of the potential depend on some *control parameter*(*s*), defined by the proper model. The change of the value of these parameters may cause the change of resilience of the system, by narrowing (widening) of the wells and/or heightening (lowering) the barrier. In what follows we will denote such a control parameter as b, and, if needed, express explicitly the dependence of force on it by $\bar{F}(\bar{X}, b)$. If the shape of potential is changing with changing value of b, thus changing stability of a given state, there may exist such value of b, for which this stable state vanishes. Such a threshold value of control parameter will be denoted by b_T.

Let us proceed to the next measure of resilience called a *return time* (or, its inverse, a *recovering rate*). It measures time needed to return to the stable "bottom" of the potential after being pushed out by a some perturbation. It what follows we will restrict ourselves to the case of one-dimensional continuous systems.

One-dimensional differential equation with explicit dependence on control parameter reads:

$$\frac{dX}{dt} = F(X, b),$$

and stationary point is defined as:

$$\frac{dX^*}{dt} = F(X^*, b) = 0. \tag{9}$$

Following Wissel [26], let us consider a small perturbation from a stationary state, $X = X^* + \delta X$, and examine its time evolution. In the vicinity of a stationary state one may linearize (9):

$$\frac{d(X^* + \delta X)}{dt} = F(X^*, b) + \frac{\partial F(X^*, b)}{\partial X}\delta X.$$

As $F(X^*, b) = 0$ the solution for δX is:

$$\delta X = (\delta X)_0 e^{\lambda t} \equiv (\delta X)_0 e^{-t/T_R},$$

with $(\delta X)_0$ being the initial perturbation and $-\frac{1}{T_R} = \lambda = \frac{\partial F}{\partial X}(X^*, b)$, where T_R is the characteristic return time. This quantity is meaningful only for $\lambda < 0$, as for $\lambda > 0$ the stationary point becomes unstable. Thus, a deviation of X from the (stable) equilibrium X^* will return exponentially in the course of time with the characteristic return time T_R.

Let us examine, how this characteristic return time changes when the control parameter b approaches the threshold b_T. It holds at the threshold: $\frac{\partial F}{\partial X}(X^*, b_T) = 0$. Let us introduce a small deviation:

$$y = X - X^*(b_T).$$

Expanding F in terms of y and $\delta b = |b - b_T|$ gives:

$$F(X, b) = F(X^*(b_T), b_T) + \frac{\partial F}{\partial X}(X^*(b_T), b_T)y + \frac{1}{2}\frac{\partial^2 F}{\partial X^2}(X^*(b_T), b_T)y^2 + \frac{\partial F}{\partial b}(X^*(b_T), b_T)\delta b.$$

As the two first terms on the right hand side of the above equation vanish, we have:

$$F(X,b) = \alpha y^2 + \beta\delta b, \tag{10}$$

where:

$$\alpha \equiv \frac{1}{2}\frac{\partial^2 F}{\partial X^2}(X^*(b_T), b_T)$$

$$\beta \equiv \frac{\partial F}{\partial b}(X^*(b_T), b_T).$$

From this, one can write for y^*:

$$\alpha(y^*)^2 + \beta\delta b = F(X^*, b) = 0,$$

and

$$y^* = \pm\left(-\frac{\beta}{\alpha}\delta b\right)^{\frac{1}{2}}.$$

As

$$\lambda = \frac{\partial F}{\partial X}(X^*, b) = \frac{\partial F}{\partial y}(y^*(\delta b), \delta b)$$

and (differentiating (10)):

$$\frac{\partial F}{\partial y} = 2\alpha y$$

then, for λ in the vicinity of threshold, one obtains:

$$\lambda = 2\alpha y^*(\delta b) = \pm 2(\alpha\beta\delta b)^{\frac{1}{2}}.$$

Thus, the general law for the characteristic return time in one-dimensional system reads:

$$T_R \propto |b - b_T|^{-\frac{1}{2}}.$$

This result means that as the system approaches a threshold a disturbed system needs more time to reach an equilibrium, and predicts the form of this dependence in the vicinity of the threshold.

Now let us proceed to apply above described measures of resilience to Brock-Durlauf model. The control parameter here will be an external influence, h. We will examine the degree of resilience of a stationary state with negative mean choice and fixed values of β and J ($\beta = 2, J = 1$), while changing value of h.

The increase of value of h causes deepening right well and, on contrary, makes the state corresponding to negative mean value more and more shallow and narrow. Fig.6 shows the dependence of width of left well (on the left) and height of the barrier (on the right) on value of h. With increasing h the left well is getting more and more narrow and the barrier is lower, until final disappearance of the left well at $h = +0.2664$.

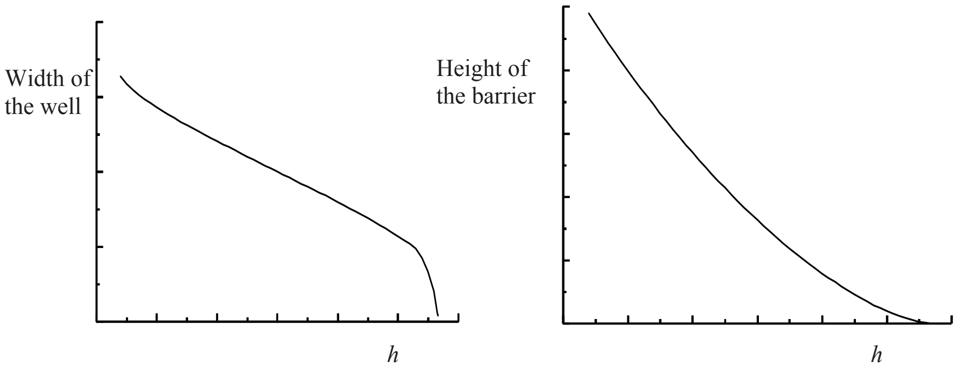

Fig. 6. Sizes of potential well in Brock-Durlauf model.

As depicted in Fig.7, the characteristic return time T_R increases with h approaching the threshold value $h_T = +0.2664$ (marked with the vertical dashed line), very slowly far from the threshold and rapidly in its vicinity. The square of the recovery rate T_R^{-2} falls to zero at the threshold, but its linear dependence on $|h - h_T|$ predicted by theory (cf. Eq. (2)) can be observed only very close to h_T. The values of β and J are these same as on Fig.6 ($\beta = 2, J = 1$).

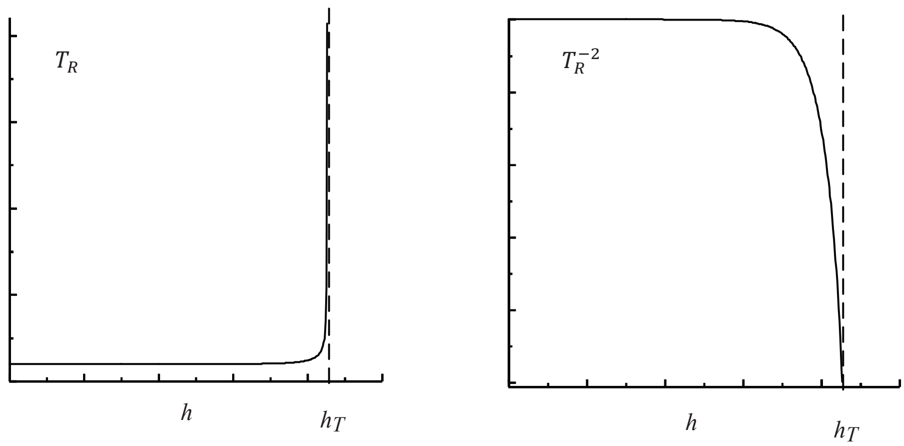

Fig. 7. Characteristic return time and square of the recovery rate in Brock-Durlauf model.

The results of our attempts to correlate the following measures of resilience: T_R, T_R^{-1} and T_R^{-2}) with the well width and the barrier height are shown in Fig.8. Monotonic dependences between these measures can be observed.

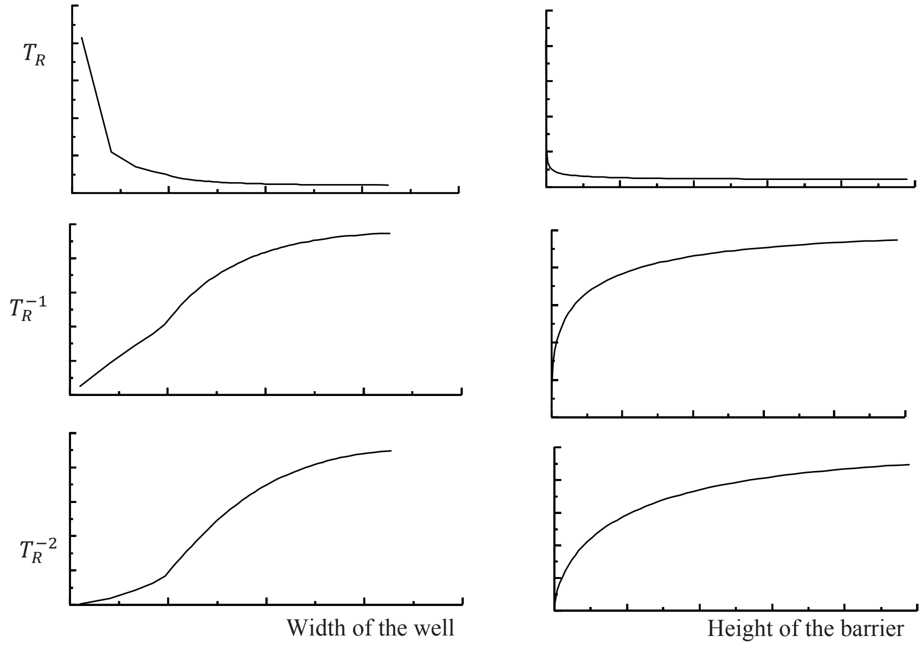

Fig. 8. Dependences between some measures of resilience in Brock-Durlauf model.

Although our result is far from being the prove of applicability of so different quantities to measure the same quality (i.e. resilience) it may be treated as a corroboration of suggestion, that the behavior of one of them can be a good predictor of behavior of the others.

4. Summary and conclusions

Growing interest in quantifying stability properties and persistence abilities of ecological and sociological systems gave rise to many attempts to measure these properties in both models and real systems. Recently, a concept of "resilience", introduced by Holling [8], has been increasingly applied in various areas of research to describe numerous kinds of systems: ecological [14, 15, 25], sociological [1], economical [6], socio-ecological [9, 24], socio-economic [12], ecological-economic [18]; and even in urban sciences (planning) to describe properties of cities [19]. Since it has been used in so many contexts and defined in so many ways that the very meaning of "resilience" gets increasingly vague and unspecified [2].

As a result attempts to add rigor appeared. There are attempts to define specific measures of this quantity: either in a strict mathematical way (for models and, at least in principle, for real systems) or as certain kinds of quantitative indicators (for real systems). There exist numerous experimental evidences that an impending regime shift is signaled by a rise of spatial and/or time variance (e.g., [10, 16, 23]). Although the variance component is difficult to distinguish from environmental noise, there are methods that allow for it and do not require detailed knowledge about mechanisms underlying the regime shift [4]. In addition to changes in values of variance, it was also observed that in the vicinity of a threshold, the power spectrum of the overturning becomes "redder", i.e., more energy is contained in the low frequencies [11].

We have mentioned here only a few, most important and frequently used, measures of resilience. We have tried them on a simple socio-economical model. It occurred, that their mutual dependences show strict monotonic character, what suggests, that all of them are proper measures of resilience and good predictors of ongoing breakdown of the system.

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Streszczenie

Koncepcja *resilience* i jej zastosowanie w pewnym socjologiczno-ekonomicznym układzie

Ważną cechą wielu układów, zarówno społecznych, jak i ekologicznych, jest ich *resilience* (stabilność, elastyczność), czyli zdolność do trwania w określonym stanie (określonym jakościowo pod pewnymi, makroskopowymi, względami), pomimo zmian niektórych parametrów oraz czynników zewnętrznych. Ocena stabilności jest niezwykle istotna, gdyż umiejętność przewidywania, który układ ekologiczny czy społeczny znajduje się na krawędzi załamania, pozwoli na lepszą jego ochronę przed potencjalną katastrofą. W pracy przedstawionych zostało kilka sposobów pomiaru zdolności układów do przetrwania, które mogą być zastosowane do modeli symulujących różne zjawiska. Używanie tych miar zostało zaprezentowane na przykładzie konkretnego modelu. Jest to model Brocka-Durlaufa, model binarnego wyboru, w którym jednostki w swoich kolejnych decyzjach kierują się zarówno korzyścią ekonomiczną, jak i konformizmem (chęcią naśladowania innych). Choć badanie właściwości modeli i ich stabilności pomaga w wypracowaniu intuicji dotyczącej zmian zdolności rzeczywistych układów do przetrwania w określonym stanie, w pracy wspomniano również o istniejących wskaźnikach, które bezpośrednio mogą służyć do przewidywania nadchodzącego załamania w realnym świecie.