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A Note of the Identification of the Bandwidth for the Potential Redistribution Index Evaluation

The income redistribution brought by a tax system is basically measured by the *RE* index, which is defined as the difference between the pre-tax Gini index and the post tax- one (cf. 0). In order to evaluate the "potential" vertical redistribution power of a tax system, together with unfairness which lowers the redistributive potentiality, Lambert et al. [1], [2] and [3] suggest to look for equal pre-tax incomes sets.

Making use of Gini index decomposition properties, the authors show that the redistribution index can be written as

$$RE = G_y - G_{yt} = V - H - R\,. \tag{1}$$

In expression (1) V measures the redistribution that would have occurred if equals had been treated equally: it is called as the potential vertical effect. H, the horizontal effect, is interpreted as the loss in the redistributive effect accounted for by the unequal treatment of equals. R, the reranking effect, is the loss in the redistributive effect caused by the difference in pre-tax and post-tax rankings of income units. All these effects are extensively discussed in 0 and 0.

The trouble is that, estimation of the vertical V, horizontal (H) and reranking (R) effect of a tax system would require division of population into groups of individuals with exactly the same pre-tax income. Being almost impossible to determine groups of exact pre-tax incomes in real data bases, groups of approximate or close equal pre-tax incomes should be determined: from which it derives the problem to determine "who is the equals" and this necessarily involves grouping of almost equal incomes. This problem is solved by determining contiguous groups in the pre-tax income parade, by partitioning the whole income range into equal income intervals: it follows that in this approach it results to be crucial the decision about such intervals width, as all measures in the right hand side of (1) depend on the income bandwidth. The choice of such a bandwidth

should be tackled according to optimality criterions. The main aim of this paper is look through the problem of defining groups of close equals, by considering in detail some of the suggestions proposed in the literature. In this paper the following *RE* decompositions will be considered:

1. The Aronson, Johnson, Lambert decomposition (AJL) (cf.0):

$$RE = V^{AJL} - H^{AJL} - R^{AJL}$$

2. Van de Ven, Creedy, Lambert decomposition (VCL) (cf.0):

$$RE = V^{VCL} - H^{VCL} - R^{AJL}$$

3. Urban, Lambert decomposition (UL) (cf. 0):

$$RE = V^{UL} - H^{UL} - R^{APK}$$

Van de Ven, Creedy, Lambert suggested choosing the bandwidth where V^{VCL} is maximum; their criterion was applied for V^{UL} by Kim and Lambert (cf. 0) and, analogously, it could be applied also for V^{AJL}.

Recently Vernizzi and Pellegrino [10] (VP) have suggested choosing the bandwidth which minimizes the ratio:

$$\Phi = \frac{\max\left\{\left|V^{VCL} - V^{UL}\right|, \left|V^{VCL} - V^{AJL}\right|, \left|V^{AJL} - V^{UL}\right|\right\}}{\min\left\{V^{VCL}, V^{AJL}, V^{UL}\right\}} \tag{2}$$

The rationale for (2) stays in the doubt that each of the three above reported measures consider some important aspects which should be taken into account, without being able to be exhaustive: (2) represents then a conservative compromise. In this paper it is extensively investigated how VCL and VP criterions behave in the framework of income and tax data collected by two Lower-Silesian revenue offices.

The experiment was conducted on Polish data coming from two Lower-Silesian tax offices from 2001, pooled together. This set of data contains information on income and tax paid for individuals and household resident in the Municipality of Wrocław and Wałbrzych. After deleting observations with non-positive gross income, the whole population consists in 130 494 individuals. The analysis were performed by autor's own programmes written in the "*R*" language.

In order to conduct the experiment on two different distributions, incomes where aggregated according to families and then considered both as

1. total family income (symbol – *total income*)

and as

2. per-spouse family income (symbol – *per spouse income*).

Obviously a criterion is better than another if it returns bandwidth for which we receive an estimate of vertical effect close to the true vertical effect. The trouble is that we do not know the true vertical effect so it is not quite to judge this criterions from this point of view. Even if we cannot evaluate the three V's under from their capabilities in measuring the unknown true potential redistribution index, we may at least request that their identification is obtained by a regular and smoothed function. So we can check how regular is the path which leads to optimal values according either to VCL or VP criterion.

On the basis of empirical analysis the regularity measures for VP Criterion are equal to 15.56 (minimum is 10) for per-spouse income and 22.22 for total income: they appear to be much lower than the corresponding ones for VCL criterion. For VCL criterion the stability measures are, respectively, 104.44 (per-spouse income) and 90 (total income) for V^{AJL}; when maximizing V^{UL} they are 47.78 and 76.77. In the context of the here considered distributions the bandwidth which maximizes V^{UL} appears to be more regular than that for maximizing V^{AJL}, however the bandwidth which minimizes Φ looks to be incomparable superior to both of them.

To sum up the behavior of the redistribution indexes proposed by Urban and Lambert (2008) are analysed using the real income data. The empirical evidence that derives from the data base on individuals and household resident in the Municipality of Wrocław and Wałbrzych, confirms UL's suggestion to look to other indexes than V^{VCL} as a measure of the potential redistribution: the bandwidth which maximizes V^{VCL} is too large to be considered as including close equals. When applying VCL maximization criterion to V^{UL} and V^{AJL}, as Kim and Lambert do for the former, the resulting bandwidths can be considered as containing what might actually looked at as close equals. However, if the step for the grid search is accurate, V^{UL} and V^{AJL} curves appear to be very irregular: they show several local maxima so that the absolute maximum appears to be irregular.

In order to override this problem, VP criterion can be applied. This criterion is a compromise of the three indexes and so it preserves the desirable properties that each index owns. Moreover the minimum of the ration which identifies VP optimal bandwidth appears to be quite regular.

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Abstract

A decomposition of actual redistributive effect was proposed by Aronson Johnson and Lambert in 1992, in order to evaluate the potential redistributive effect. However this decomposition is not univocally determined, but as it can be calculated after having gathered incomes into groups of "close" equals, de facto it depends on the bandwidth chosen to split the income parade into contiguous income groups; it follows that the bandwidth has to be chosen according to proper criteria: Van de Ven, Creedy, Lambert (VCL criterion) propose to choose the bandwidths where potential redistributive indexes are maxima. However the literature proposes more than one index to measure the potential redistribution of a tax system and the maxima associated to each of them do not necessarily coincide and, moreover, they do not generally show a regular sequence of values leading to the global maxima. The main aim of this paper is to contribute to the problem of defining a proper bandwidth which can split the income parade into close equal groups: VCL criterion is considered together to a minima criterion

recently proposed by Vernizzi and Pellegino (VP criterion). Empirical evidence is obtained by a data set of incomes and taxes collected by two Lower-Silesian revenue offices. The analysis were performed by autor's own programmes written in the "*R*" language.

Keywords: Redistributive effect, close equal group
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